Abstract of the thesis "Regular tree languages in the first two levels of the Borel hierarchy"

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In the thesis we focus on a quite recent field of research, that is, the connection between Descriptive Set Theory and Automata Theory. The overlap between these two areas is that in Automata Theory the space which we work with (when we work with infinite objects) is, in fact, the Cantor space 2^ω, that is a well-known uncountable Polish space. In Descriptive Set Theory subsets of the Cantor space can be stratified through topological hierarchies, like the Borel hierarchy, the Wadge hierarchy and the difference hierarchy, while in Automata Theory these spaces can also be studied in terms of "regularity", that is the property of being recognised by an automaton. This double point of view leads to many interesting questions about the interplay between topological complexity and regularity.

While we have a complete picture of these relationships in the case of automata on words, the case of automata on trees is still a *terra incognita*. Some results have been obtained for particular classes of languages, like Büchi languages, deterministic languages or unambiguous languages. But in the thesis we focus on general regular tree languages, without any restriction. We show some new results that are concerned with general regular tree languages that lie in low levels of the Borel hierarchy and the Wadge hierarchy. In particular we prove the following:

Theorem 1. *A regular tree language L is recognised by a weak-alternating automaton that uses only two priorities if and only if it is in the first level of the Borel hierarchy.*

Then we prove some results about slightly higher levels of the Wadge hierarchy that can be summed up by the following theorem:

Theorem 2. *The following holds:*

1. *Let Γ be a Wadge degree with finite Wadge rank. Then it is decidable if a regular tree language L belongs to Γ.*
2. *It is decidable if a regular tree language L is a Boolean combination of open sets.*
3. *It is decidable if a regular tree language L is in the Borel class Δ^0_2.*

All the results of this theorem are contained in [BCPS18].

Finally, we give a complete characterisation of the second level of the Borel hierarchy [CMS17]:

Theorem 3. *It is decidable if a regular tree language L belongs to the second level of the Borel hierarchy. Moreover, a regular language L is in the second level of the Borel hierarchy if and only if it is recognised by a weak-alternating automaton that uses exactly three priorities.*

References

[BCPS18] Mikołaj Bojańczyk, Filippo Cavallari, Thomas Place, and Michał Skrzypczak. Effective characterisations of regular tree languages in low levels of wadge hierarchy. *Preprint*, 2018.

[CMS17] Filippo Cavallari, Henryk Michalewski, and Michał Skrzypczak. A characterisation of Pi^0_2 regular tree languages. In *MFCS*, pages 56:1–56:14, 2017.